Lévy processes and the financial crisis: can we design a more effective deposit protection?

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Abstract

Lévy processes have been applied in various financial settings to overcome the main shortcomings of the Gaussian distribution, since they allow for fat tails and jumps. In the present paper we propose to use Levy processes to simulate the distribution of losses deriving from bank failures. The application of Levy processes is expected to provide successful results to this aim since bank failures are unexpected, rare events. We propose to use the simulated distribution of losses to design an effective Deposit Guarantee Schemes (DGSs). DGSs are financial institutions whose main aim is to provide a safety net for depositors. If a credit institution fails, depositors will be able to recover their bank deposits up to a certain limit. During the recent global financial crisis, DGSs were brought at the centre of the political and financial debate, especially due to the fact that the DGSs in the European Union Member States resulted in most of the cases incapable to react to the financial crisis, especially due to the lack of funds set aside. By simulating banks’ default and the corresponding losses, our model allows defining a target level for the funds to be collected by the scheme in order to promptly and effectively respond to financial crisis and protect the citizens. The proposed approach is applied to a sample of Italian banks.

Disclaimer: The opinions presented here are exclusively those of the authors and do not in any way represent those of the European Commission.

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1 Introduction

Deposit Guarantee Schemes (DGSs hereinafter) are financial institutions set up with the main purpose of reimbursing depositors whenever their bank goes into default. If a credit institution fails, a DGS intervenes and pays back the bank deposits up a certain amount, called level of coverage. It is clear that, in order to work properly, the DGS must have at its disposal an adequate amount to cover potential losses. This amount is usually set aside by collecting contributions from banks. It is quite well-known that the existence of these institutions leads to some benefits: from depositors’ point of view, DGSs protect a part of their wealth from bank failures and avoid bank runs; from banking stability perspective, DGSs contribute to strengthen the confidence in the financial sector, thus preventing bank runs, and to create a level playing field, thus avoiding competitive distortions (see for example Garcia (1999), Cariboni et al. (2010) and European Commission (2011)).

These schemes are in place in many countries all over the world, like in the US, Canada, Russia, and Australia (Laeven (2002) summarizes the main features of the existing DGSs in the world). In the European Union, Directive 94/19/EC (European Parliament and Council (1994)) obliged Member States to ensure the existence of at least one or more schemes on their territory, but required only minimum harmonization of rules across DGSs (for example, it required DGSs to set the minimum level of coverage at €20,000). The Directive left a large degree of discretion to the schemes, especially in relation to the financing mechanisms. The levels of coverage in place among EU DGSs ranged from around €14,000 in Latvia to around €103,000 in Italy; also the way DGSs financed themselves has been very heterogeneous. Some DGSs collected contributions from their members on a regular basis, while others called for contributions only in case of bank’s failure (see Cariboni et al. (2008) and Cariboni et al. (2010)).

In 2005-2006 the European Commission launched a review process of Directive 94/19/EC, the conclusions of which were disseminated via a formal Communication in 2006 (European Commission (2006)). The Commission highlighted a number of short-term improvements to the existing arrangements, to be adopted via self regulatory agreements and without changing the legislation in place. The improvements included fine tuning topping-up arrangements\(^1\), shortening the time it takes for schemes to pay out to depositors after a bank’s failure and developing exchange of information between schemes.

The 2008 global financial crisis brought DGSs at the centre of the political and financial debate. In order to restore confidence in the financial sec-

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\(^1\)They are arrangements where a bank branch in another Member State voluntarily joins the host country’s DGS.
tor, in October, 2008, the Commission proposed urgent legislative changes of the Directive (European Commission (2008)). The Amending Directive, adopted in March 2009 (European Parliament and Council (2009)), compelled EU schemes to increase the level of coverage from €20,000 to €50,000 first and to €100,000 by December, 2010. Moreover, it obliged European DGSs to reduce the maximum time necessary to repay depositors from 3 months to 20 working days and to discontinue coinsurance. The Amending Directive was, however, only an emergency measure aiming at maintaining depositors’ confidence in the financial system.

One of the consequences of the financial crisis was that it emphasized the necessity of an in-depth revision of the whole Directive on DGSs. As a result, in July, 2010, the Commission adopted a legislative proposal on DGSs (European Commission (2010)). This proposal would aim at simplifying and harmonizing many aspects of the functioning left to the discretion of DGSs up to now. The aspects mentioned in the proposal which will be more relevant for the present work are the following:

- Simplification and harmonization of the scope of coverage. Only deposits by customers and by non-financial corporations would be eligible for protection in all DGSs.

- Harmonization of the financing mechanisms of DGSs. All DGSs would have to move to an ex-ante financing system, where financial resources are collected from member banks in advance on a regular basis.

- Choice of the target level for the funds of DGSs. The target level for the funds would be fixed equal to 2% of the amount of deposits eligible for protection. The transition period to let DGSs reach the target level would be equal to 10 years.

The academic literature on DGSs can be divided into two groups, depending on the way the default event is defined (the paper by De Lisa et al. (2010) provides a comprehensive summary of the existing literature). Few studies (Duffie et al. (2003) among the others) adopt reduced-form models to estimate fair market premiums, while most studies (Bennett (2002), Kuritzkes et al. (2002) and Sironi and Zazzara (2004) for example) estimate banks’ default probabilities from market data and relying on structural credit risk models. The new model, recently developed by De Lisa et al. (2010), proposes a novel approach to estimate loss distributions which explicitly considers the link between deposit insurance and the regulatory framework for capital requirements introduced by Basel II.

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2Directive 94/19/EC allowed DGSs an optional coinsurance of up to 10%, i.e. a certain percentage of losses was borne by depositors.

3According to the current practices, DGSs may provide that certain classes of deposits, detailed in Annex I of Directive 94/19/EC (European Parliament and Council (1994)), shall be excluded from protection.
The main aim of this paper is to design an effective DGS. In particular our model allows defining a target level for the funds to be collected by the scheme in order to promptly and effectively respond to financial crisis and protect the citizens. To achieve this goal we simulate banks’ defaults and the corresponding losses potentially hitting the system. This approach is applied to a sample of Italian banks.

Following a well recognized approach (Bennett (2002), Kuritzkes et al. (2002) and Sironi and Zazzara (2004) among the others), funds can be regarded of as portfolios of counterparty risks. These portfolios consist of individual exposures to insured banks, each of which has a small but non-zero probability of causing a loss to the fund. We simulate the empirical loss distribution of the DGS to investigate whether the proposed target size is adequate to face potential banks’ failures.

The procedure adopted to simulate the loss distributions relies on the classical credit risk techniques (see Bluhm et al. (2003), Schönbucher (2003) and Schoutens and Cariboni (2009)): defaults occur if the bank’s asset value falls below a threshold, asset-value processes follow a generic one-factor model and default times are exponentially distributed. A novel approach is proposed to model asset-value processes and to estimate banks’ default probabilities. Asset-value processes are assumed to follow a generic one-factor Lévy process and in particular the one-factor Gaussian model and the one-factor Shifted Gamma Lévy model are investigated. Moreover, a novel approach is proposed to estimate banks’ default probabilities, which are inferred from CDS spreads, assuming an underlying pricing model. This procedure is quite common in literature, but to our knowledge it has never been explored in the context of DGS. We also study linear models to link the default probabilities estimated from CDS spreads to a set of financial indicators and we apply this model to a larger sample of banks; in this way we come to an estimate of the default probability also for those banks which do not underlie a CDS contract.

The approach is applied to a sample of 51 Italian banks accounting for around 60% of the total amount of eligible deposits and for around 43% of total assets as of 2006 in Italy.

This paper is organized as follows: Section 2 introduces the main features of the functioning of DGSs and gives an overview of the existing scientific literature on DGS; Section 3 describes the methodology applied to build the empirical loss distributions; Section 4 presents the results and Section 5 concludes.
2 Summary of the current state of art

2.1 Main features of Deposit Guarantee Schemes

In this paragraph we define some key concepts related to DGSs used in the remainder of this paper.

The level of coverage is the level of protection granted to deposits in case of default. If a bank fails, the scheme repays deposits only up to a certain amount, which is equal to the level of coverage. The scheme has to pay for every deposit an amount equal to

$$\min \{\text{Amount of deposit, Level of coverage}\}.$$ 

In this paper we will work with two types of deposits: eligible deposits and covered deposits. Eligible deposits are those deposits eligible for protection by DGSs, i.e., all those classes of deposits which are entitled to be reimbursed by the scheme in case of failure, before the level of coverage is applied. The EU Directive fixes which deposits are not entitled to be protected (Article 2 of Directive 94/19/EC). DGSs are, however, allowed to choose which classes of deposits they protect among those listed in Annex I of Directive 94/19/EC. According to the proposal adopted by the European Commission (2010), only deposits by customers and by non-financial corporations would be eligible for protection.

Covered deposits are the amount of deposits obtained from eligible deposits when applying the level of coverage: this is the amount to be effectively paid by the scheme in case of failure.

The following example will clarify the relationships between the above elements. Consider a DGS with only one member bank. The bank has 3 deposits, A, B, and C. Suppose the sizes of the deposits are, respectively, €85,000 for deposit A, €75,000 for deposit B, and €20,000 for deposit C. Moreover, suppose that deposit A is not eligible for protection, and that the level of coverage is €50,000. The amounts of eligible and covered deposits are:

$$\text{Eligible deposits} = \text{€}(75,000 + 20,000) = \text{€}95,000$$
$$\text{Covered deposits} = \text{€}(50,000 + 20,000) = \text{€}70,000.$$ 

Most DGSs in Europe collect contributions from their member banks in advance, on a regular basis: these contributions fill up the fund the scheme sets aside and will be employed in case of intervention to repay the deposits.

2.2 Literature Review

Up to our knowledge, the first mathematical model applied to DGSs has been developed by Merton in his seminal paper (Merton (1977), based on Merton (1974)) and it has subsequently been implemented by Markus and Shaked (1984) and by Ronn and Verma (1986). In this model the fair
premium each bank should pay to the scheme is computed by treating the deposit insurance as a put option written on the bank’s asset value.

Later on, two main groups of models have been developed. They were based on two different credit risk models: the reduced-form models and the structural credit risk models (for a comparison of the two models see, for example, Jarrow and Protter (2004)).

The first class of models defines defaults as stopping times, whose intensities depend upon financial and macroeconomic conditions. The model developed by Duffie et al. (2003) for example applies methods for the pricing of fixed-income securities subject to default risk to compute the fair premia that banks should pay. The fair premium for each bank is a function of its short-term credit spread, the expected loss at failure per dollar of protected deposits and the expected loss given default on the bank’s debt.

The second class of models considers defaults as events occurring when the bank’s asset value falls below a certain threshold, usually correspondent to its liabilities’ value.

The model first presented by Bennett (2002) and then reappraised by Kuritzkes et al. (2002) aims at building an empirical fund’s loss distribution faced by the American DGS (Federal Deposit Insurance Corporation, FDIC hereinafter), as loss distribution can be used to determine the appropriate level of fund adequacy. Banks’ default probabilities are modeled according to the Vasicek (2002) model and loss distributions are built by running Monte Carlo simulations. Bennett (2002) investigates some possible techniques to estimate banks’ default probabilities. The first technique relies upon internal models which translate credit ratings into default probabilities. Moreover, logit models are investigated: in these models the log of the odds-ratios is assumed to be linearly related to a number of financial indicators, covering capital adequacy, asset quality, earnings and safety-and-soundness areas. In the paper by Kuritzkes et al. (2002) default probabilities are estimated from credit ratings of banks provided by Moody’s or Standard & Poor’s and from the internal credit scoring model developed by FDIC.

Sironi and Zazzara (2004) apply a similar approach to the 15 largest Italian listed banks. They estimate the empirical loss distribution and compute risk-based premia that take into account contributions to both scheme’s fund expected and unexpected losses. Default probabilities are estimated from Moody’s KMV model (see Crosbie (1999)).

Instead of building an empirical loss distribution using Monte Carlo simulations, Dev et al. (2006) developed an analytical model to determine the appropriate size of the scheme’s fund and the premia banks should pay. They explicitly take into account the banks’ liability structure, especially distinguishing between insured and uninsured deposits, and claims senior to deposits.

The model proposed by De Lisa et al. (2010) explicitly considers the link that exists between deposit insurance and the regulatory framework
for capital requirements introduced by Basel II. A bank goes into default if its obligor’s losses exceed its actual capital, which is given by the Basel II regulatory capital plus the excess capital, if any. Banks’ default probabilities and the corresponding losses are computed according to the Basel II FIRB (Foundation Internal Rating Based) formula and by making use of publicly available regulatory capital information. The impact of systemic risk is included in the model via two sources. The first source depends on the fact that banks have correlated exposures and thus common exposures to the business cycle. The second source depends on the domino effect across the banking system due to linkages between banks produced by the interbank lending market.
3 Research Methodology

In the light of the recent financial crisis, the appropriateness of the size of a fund of a DGS and the definition of the amount of banks’ contributions have become a core topic. One of the key issues in the recent scientific research literature (Campolongo et al. (2010)) is to assess what would be the adequate size of the fund that a DGS should set aside.

It is straightforward to recognize that the DGS-fund can be regarded as portfolios of counterparty risks, as already highlighted, for example, in the papers by Bennett (2002), Kuritzkes et al. (2002), and Sironi and Zazzara (2004). These portfolios consist of individual exposures to insured banks, each of which has a small but non-zero probability of causing a loss to the fund; in general there is a high probability of a small loss to the fund, but there is also a (small) positive probability that the fund will incur large losses stemming from a single large bank failure or from the simultaneous failure of a large number of banks.

Despite the similarities between the DGS-funds and a portfolio of loans, it is clear that the default events are different. The defaults on individual loans simply occur when the borrower is unable to afford its payments, while banks fail because of a combination of credit, market and operational risks. Moreover, the failure of a bank is not a sudden event, but is a regulatory one, because only supervisory authorities can declare the default of an institution.

In the following we will describe the methodology adopted to build the empirical loss distribution of the fund. The empirical loss distribution has a twofold scope:

- To assess the current level of security (represented by the distribution’s percentile) provided to deposits by DGS current financial endowments (see Campolongo et al. (2010)).
- To choose a proper target size for the fund such that it can afford a desired level of protection (the target fund is fixed in a way such that it provides protection up to the desired percentile).

The methodology relies on the four following steps:

1. Estimate banks’ default probabilities from CDS spreads market data where available and from risk indicators elsewhere and calibrate the default intensities of the default time distributions;

2. Draw realizations of the asset value process;

3. From the asset values’ draws compute the the corresponding default times;

4. Evaluate the corresponding losses and compare it with the available DGS-fund.
This approach is applied to a sample of 51 Italian banks accounting for around 60% of the total amount of eligible deposits and for around 43% of total assets as of 2006 in Italy.

3.1 Estimating the banks’ default probabilities

We propose to estimate banks’ default probabilities from the corresponding CDS market data because the CDS premia are among the best measures of the market pricing of credit risk currently available. This is mainly due to standardized contracts and the relatively high liquidity in the market (Raunig and Scheicher (2009)). Unfortunately, CDS contracts are written only on a very limited number of banks: in 2006, our reference year, CDS contracts were written on only around 40 European banks, of which only 4 Italian banks.

In order to enlarge our sample, we make use of the entire set of European banks underlying a CDS contract to investigate possible relations between default probabilities and the set of financial indicators mentioned in the proposal (see European Commission (2010)) to compute risk-based contributions. This relation could then be applied to those institutions which do not have a CDS contract.

In developing this approach, particular attention should be paid to the differences between the risk-neutral and the historical default probabilities (labeled \( DP^Q \) and \( DP^P \) respectively). Mathematically speaking, the risk-neutral probability is the probability measure under which the current market price of a generic contingent claim is equal to the discounted expected value of its future cash flows (Björk (1998)). The corresponding risk-neutral default probabilities are used for pricing because they build an extra return, called risk premium, to compensate market participants for the risk they are bearing (Hull et al. (2005)). Historical default probabilities are probabilities calculated from historical data, and they are not used for pricing purposes.

The two probabilities settings lead to different default probabilities estimates. Historical default probabilities are usually smaller than risk-neutral ones because the latter probabilities reflect the risk premia required by market participants to take on risks associated with default (Duffie and Singleton (2003)).

According to literature practices, risk-neutral default probabilities are inferred from prices available on the markets, by assuming some underlying pricing structure. Historical default probabilities are used when evaluating and building relationships between default probabilities and economic/financial indicators.

In this paper we deal with both default probabilities, depending on the set of available data for each bank. The following steps are applied to

\(^4\)Data on total assets have been gathered from European Central Bank publications.
estimate the default probabilities:

1. Estimate European banks risk-neutral default probabilities from CDS spreads for the sample of European banks;

2. Using data on the European banks underlying CDS contracts, calibrate a map between risk neutral and historical default probabilities;

3. Using data on the European banks underlying CDS contracts, estimate a model between risk indicators and historical default probabilities;

4. Using data on the European banks underlying CDS contracts, apply the model estimated in step 3 to the sample of Italian banks in order to get an estimate of their historical default probabilities;

5. Estimate risk-neutral default probabilities by applying the reverse map mentioned in step 2.

The steps listed above are summarized in Figure 1. The risk-neutral default probabilities will be used to calibrate the term structure of the banks’ default probabilities.

![Figure 1: Procedure for the estimation of banks’ probabilities](image)

3.1.1 Estimating the banks’ default probabilities from CDS

A Credit Default Swap (for a detailed description refer to Duffie and Singleton (2003) and to Schoutens and Cariboni (2009)) is an over the counter
bilateral agreement where the protection buyer transfers the credit risk of a reference entity to the protection seller for a determined amount of time $T$. The buyer of this protection makes predetermined payments to the seller. The payments continue until the maturity date $T$ of the contract, or until default occurs, whichever is earlier. In the case of default of the reference entity, the protection seller pays to the protection buyer a determined amount. The CDS spread $c$ is the yearly rate paid by the protection buyer to enter a CDS contract against the default of a reference entity, reflecting the riskiness of the underlying credit.

Given the recovery rate $R_i$ and the discount factor, the CDS spread $c_i$ is a function of the default probability. We assume in our model that the default time of the $i$-th bank $\tau_i$ is exponentially distributed with intensity parameter $\lambda_i$. We consider the cumulative risk-neutral default probability $p(t)$, which is the risk-neutral probability that default will occur in $[0, t]$ (Schoutens and Cariboni (2009)): the corresponding term structure of the cumulative risk-neutral default probability for the $i$-th bank, $p_i(t)$, has the following expression:

$$p_i(t) = 1 - e^{-\lambda_i t}.$$  \hfill (1)

It can be easily shown (see Duffie and Singleton (2003), Schönbucher (2003) and Schoutens and Cariboni (2009)) that the spread is equal to:

$$c_i = (1 - R_i) \lambda_i.$$  \hfill (2)

In this work we make use of the 2006 daily 5 years-CDS spreads of 40 European banks, provided by Bloomberg\(^5\); we also assume a recovery rate $R_i$ constant for all banks and equal to 40%.

### 3.1.2 Building a map between risk-neutral and historical probabilities

Following Hull et al. (2005), we estimate the historical default probabilities from statistics on average cumulative global default yearly rates published by Moody’s (Emery et al. (2008)). In its annual reports, Moody’s provides estimates of the yearly historical firms default probabilities grouped by rating classes. Starting from these data, we estimate the corresponding rating classes’ historical default probabilities. We then associate every rating class (and every historical default probability) with a risk-neutral default probability as follows. Given a rating class, we consider all the banks (among those in our sample of European banks underlying a CDS) belonging to that class, and we associate that rating with a risk-neutral default probability equal to the average risk-neutral default probability of all the banks in that class. For example, focusing on the rating class Aaa, the associated

\(^5\)Bloomberg has been accessed from Bocconi University, 19\textsuperscript{th} November 2010.
risk-neutral default probability $DP^Q_{\text{Aaa}}$ is computed as follows:
\[
DP^Q_{\text{Aaa}} = \frac{1}{n_{\text{Aaa}}} \sum_{i \in \text{Aaa}} DP^Q_i,
\]
where $n_{\text{Aaa}}$ is the number of banks with a rating score equal to Aaa and $DP^Q_i$ is the risk-neutral default probability of the $i$-th bank with the given rating (we gathered rating scores for 36 out of 40 European banks from Moody’s web-site). This procedure let us attain the one to one correspondence between the historical and risk-neutral default probabilities reported in Table 1.

<table>
<thead>
<tr>
<th>Rating</th>
<th>Aaa</th>
<th>Aa1</th>
<th>Aa2</th>
<th>Aa3</th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$DP^Q$</td>
<td>0.0975%</td>
<td>0.1196%</td>
<td>0.1265%</td>
<td>0.1558%</td>
<td>0.1976%</td>
<td>0.3053%</td>
<td>0.4957%</td>
</tr>
<tr>
<td>$DP^P$</td>
<td>0.0022%</td>
<td>0.0038%</td>
<td>0.0067%</td>
<td>0.0116%</td>
<td>0.0201%</td>
<td>0.0348%</td>
<td>0.0604%</td>
</tr>
</tbody>
</table>

Table 1: One-to-one correspondence between historical and risk-neutral default probabilities. Data sources: Emery et al. (2008), Moody’s and Bloomberg

From the one-to-one correspondence we want to infer a continuous and closed form map that allows us move from one probability measure to the other:
\[
DP^P = f(DP^Q), \quad f : [0, 1] \rightarrow [0, 1].
\]
The function $f(\cdot)$ must be convex because $DP^Q \geq DP^P$ and must satisfy the constraint $f(0) = 0^6$ (see Berg (2010)). A suitable expression for $f$ is thus
\[
f(x) = e^{x^n} - 1. \quad (3)
\]

We calibrate the model by minimizing the Root Mean Square Error: according to this procedure, the optimal parameter is $a = 1.39$. Figure 2 presents the results of the calibration exercise. The blue dots are the points in Table 1. The continuous line is the map of Equation (3) calibrated on the same set.

Using the calibrated map $f$, we can estimate the historical default probabilities of the 40 European banks in our sample.

It is important to stress why we want to move from the risk-neutral to the historical probability measures. According to literature practices (Chan-Lau (2006)), we use risk indicators, built from balance sheet variables, to estimate banks’ default probabilities. Balance sheet data are backward looking by construction, and they give information only on what has happened

\[6\text{We should also take into account the second constraint } f(1) = 1. \text{ As our data are all close to zero and default probabilities will be unlikely to assume values close to one, we relax the second constraint.}\]
in the past (Huang et al. (2009)). If we want to use these variables to get banks’ default probabilities estimates, the correct probability to be associated to them is the historical default probability, because it is a backward looking probability measure. Risk-neutral probabilities, on the contrary, are forward looking measures and thus this choice would be incongruous with the backward looking behavior of risk indicators based on balance sheet data.

3.1.3 Building a map between risk indicators and historical default probabilities

We investigate linear models

\[ DP^P = X\beta + \epsilon \]  

between the historical default probabilities estimated in Section 3.1.2 and a set of financial (risk) indicators. In literature there exists a number of possible financial indicators; in this paper we have restricted our attention to the risk indicators mentioned in the proposal adopted by the European Commission (2010). As liquidity indicators are left to the discretion of the single Schemes, we have taken into account those suggested by the Joint Research Centre of the European Commission (2009). Risk indicators and balance sheet data as of 2006 have been gathered by Bankscope database\(^7\).

Among all possible choices of indicators, the set of indicators that best explains the \( DP^P \) is the one listed in Table 2. The \( R^2 \) coefficient of this

\(^7\)Bankscope has been accessed from Bocconi University, Milan, 19\(^{th}\) November 2010.
Table 2: Financial indicators for the regression model. Data source: Bankscope.

<table>
<thead>
<tr>
<th>ROAA</th>
<th>Exc. Capital</th>
<th>Total Assets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liquid Assets</td>
<td>Exc. Capital</td>
<td>Risk-weighted Assets</td>
</tr>
<tr>
<td>Customer &amp; ST Funding</td>
<td>Loan Loss Provisions</td>
<td>Net Interest Revenue</td>
</tr>
<tr>
<td>Net Loans</td>
<td>Loan Loss Provisions</td>
<td>Operating Income</td>
</tr>
<tr>
<td>Customer &amp; ST Funding</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cost to Income</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Regression is 50.78% and the corresponding p-value is 5.2%.

3.1.4 Estimating historical default probabilities

We now focus on the sample of 51 Italian banks. Starting from 2006 data on risk indicators, we apply the model above estimated (Equation (4)) in order to get an estimate of the historical default probabilities.

3.1.5 Estimating risk-neutral default probabilities

We apply the reverse map described by Equation (3) to the previously estimated historical default probabilities and we get an estimate of risk-neutral default probabilities for the 51 Italian banks. Under this assumption, from the estimated risk-neutral default probabilities we get default intensity $\lambda_i$ estimates:

$$\lambda_i = -\ln (1 - DP_i^Q).$$

Default intensity parameters are among the inputs of the loss distribution’ simulation (see Section 3.3).

3.2 Simulating banks’ defaults

In order to build the empirical loss distribution of the scheme’s fund, we must define what we mean by default. In the general case of a credit portfolio, a loss occurs if a borrower defaults. In the specific case of a Deposit Guarantee Scheme, a loss occurs if an insured bank fails, thus triggering a fund’s payout, as assumed for example by Sironi and Zazzara (2004). In particular we assume that a single bank goes into default when its asset value falls below a certain threshold. In the following we propose two approaches to model the bank’s asset value, both based on generic one-factor
Lévy models: the Gaussian one-factor model and the Shifted Gamma Lévy model (see Schönbucher (2003) and Albrecher et al. (2006)).

3.2.1 Generic One-Factor Lévy Model

In this paragraph we first shortly present Lévy processes and then we introduce the generic one-factor Lévy model.

Let \( \phi(z) \) be the characteristic function of a distribution. If, for every positive integer \( n \), \( \phi(z) \) is also the \( n \)-th power of a characteristic function, we can say that the distribution is infinitely divisible. One can define for every infinitely divisible distribution a stochastic process \( X = \{ X_t, t \geq 0 \} \), called Lévy process, which starts at zero, has stationary and independent increments and such that the distribution of an increment \( X_{t+s} - X_s \), with \( s, t \geq 0 \) has \( (\phi(z))^t \) as characteristic function (see Schoutens (2003), Cont and Tankov (2004) and Schoutens and Cariboni (2009) for more details about the applications of Lévy processes in finance).

The function \( \psi(z) := \log \phi(z) \) is called the characteristic exponent and it satisfies the following Lévy-Khintchine formula

\[
\psi(z) = i\gamma z - \frac{\varsigma^2}{2} z^2 + \int_{-\infty}^{\infty} \left( e^{izx} - 1 - izx1_{|x|<1} \right) \nu(dx), \quad z \in \mathbb{R},
\]

where \( \gamma \in \mathbb{R}, \varsigma^2 \geq 0 \), and \( \nu \) is a measure on \( \mathbb{R} \setminus \{0\} \) such that

\[
\int_{-\infty}^{\infty} (1 \wedge x^2) \nu(dx) < \infty.
\]

From the Lévy-Khintchine formula one sees that, in general, a Lévy process consists of three independent parts: a linear deterministic part, a Brownian part, and a pure jump part. The corresponding infinitely divisible distribution is said to have a Lévy triplet \([\gamma, \varsigma^2, \nu(dx)]\).

The measure \( \nu \) is called the Lévy measure and it dictates how the jumps occur: jumps of sizes in a set \( A \) occur according to a Poisson process with parameter \( \nu(A) = \int_A \nu(dx) \): \( \nu(A) \) is thus the expected number of jumps per unit of time whose size belongs to \( A \).

Now we turn to the generic one-factor Lévy model. Let us choose an infinitely divisible distribution \( L \) and let \( X = \{ X_u, u \in [0,1] \} \) and \( X^{(i)} = \{ X_u^{(i)}, u \in [0,1] \}, i = 1, \ldots, M \) be independent and identically distributed Lévy processes such that \( X_1 \) and \( X_1^{(i)}, i = 1, \ldots, M \) follow the law \( L \). We further assume that \( \mathbb{E}[X_1] = 0 \) and \( \text{var}(X_1) = 1 \) and from this it can be shown that \( \text{var}(X_u) = u \).

Let \( \rho \in (0,1) \); the generic one-factor Lévy model for the asset value of the \( i \)-th bank, \( i = 1, \ldots, M \) at time \( t \) is of the form

\[
A_i(t) = X_\rho + X_{1-\rho}^{(i)}, \quad i = 1, \ldots, M.
\]  

(5)
Each \( A_i(t) \) has by the stationary and independent increments property the same distribution \( L \) with distribution function \( F_{X_1} \), where \( F_{X_u} \) is the distribution function of \( X_u, u \in [0,1] \). As a consequence, \( \mathbb{E}[A_i(t)] = 0 \) and \( \text{var}(A_i(t)) = 1 \). Furthermore, it can be easily proved that the asset values of any two banks \( i \) and \( j \), with \( i \neq j \), are correlated with linear correlation coefficient \( \rho \).

The default probability term structure for all banks \( p_i(t) \), \( 0 \leq t \leq T \) is known (see Section 3.1.1 and Section 3.1.5). The \( i \)-th bank defaults at time \( t \) if the asset value \( A_i(t) \) falls below a determined threshold \( K_i(t) \). In order to match default probabilities under this model with the term structure of default probabilities defined in Equation (1), we have to set \( K_i(t) = F_{X_1}^{(-1)}(p_i(t)) \). It follows that:

\[
P [ A_i(t) \leq K_i(t) ] = P \left[ A_i(t) \leq F_{X_1}^{(-1)}(p_i(t)) \right] = F_{X_1} \left( F_{X_1}^{(-1)}(p_i(t)) \right) = p_i(t).
\]

From the above relationship, the default time \( \tau_i \) of the \( i \)-th bank then equals:

\[
\tau_i = p_i^{(-1)} (F_{X_1}(A_i)) = -\frac{\ln(1 - F_{X_1}(A_i))}{\lambda_i},
\]

where \( \lambda_i \) is the default intensity of the default probability term structure (Equation (1)), and \( A_i \) is a realization of the asset value process \( A_i(t) \).

### 3.2.2 One-factor Gaussian model

The Brownian motion is a particular Lévy process with Lévy triplet \( [\mu, \sigma^2, 0] \). The generic one-factor Lévy model built with a Brownian motion coincides with the Vasicek (2002) model. The generic one-factor Lévy model in this case can be rewritten as:

\[
A_i(t) = \sqrt{\rho} Y + \sqrt{1 - \rho} X_i, \quad i = 1, \ldots, M,
\]

where \( Y \) and \( X_i \) \( i = 1, \ldots, M \) are independent normally distributed random variables with zero mean and variance 1, \( \rho \in (0,1) \) and \( M \) is the number of banks (see Schönbucher (2003)). The variable \( Y \) is a common factor and can be interpreted as a systematic risk factor, common to all banks, while \( X_i \) is an idiosyncratic noise component and it represents the firm specific risk factor; the parameter \( \rho \) is the correlation factor. This model assumes that the vector of \( M \) asset values \( A_i(t) \) is multivariate standard Normal distributed:

\[
[A_i(t)] \sim \mathcal{N}(0, \Sigma), \quad \text{where } \Sigma_{ij} = \begin{cases} 1 & i = j \\ \rho & i \neq j \end{cases}.
\]

In this case the default time \( \tau_i \) of the \( i \)-th bank equals:

\[
\tau_i = p_i^{(-1)} (\Phi(A_i)) = -\frac{\ln(1 - \Phi(A_i))}{\lambda_i},
\]
where Φ is the cumulative distribution function of the standard Normal distribution, λ_i is the default intensity of the default probability term structure (Equation (1)), and A_i is a realization of the asset value process A_i(t).

### 3.2.3 One-factor Shifted Gamma Lévy model

We recall that the density function of the gamma distribution Gamma(a,b) with a > 0, b > 0 has the following expression:

\[ f(x) = \frac{b^a}{\Gamma(a)} x^{a-1} e^{-xb}, \quad x > 0 \]

and the corresponding characteristic function is given by

\[ \phi(z) = \frac{1}{(1 - izb)^a}, \quad z \in \mathbb{R}. \]

The characteristic function is infinitely divisible.

Let us consider a unit-variance gamma process \( G = \{G_u, u \geq 0\} \) with parameters \( a > 0 \) and \( b = \sqrt{a} \) such that \( \mathbb{E}[G_1] = \sqrt{a} \) and \( \text{var}(G_1) = 1 \). As a driving Lévy process we consider the Shifted Gamma process:

\[ X_u = \sqrt{au} - G_u, \quad u \in [0,1]. \tag{9} \]

The (financial) interpretation in terms of asset value is that there is a deterministic up trend, given by \( \sqrt{au} \) with random downward shocks \( G_u \) (see Albrecher et al. (2006)).

The one-factor shifted Gamma Lévy model is given by Equation (5)

\[ A_i(t) = X_\rho + X_{1-\rho}^{(i)}, \quad i = 1, \ldots, M, \]

where \( X_\rho \) and \( \{X_{1-\rho}^{(i)}, \rho \in (0,1)\} \ i = 1, \ldots, M \) are independent standardized Shifted Gamma processes, defined as

\[ X_\rho = \sqrt{a\rho} - G_\rho \quad \text{and} \quad X_{1-\rho}^{(i)} = \sqrt{a(1 - \rho)} - G_{1-\rho}. \]

By construction, each \( A_i(t) \) has the same distribution as \( X_1 \).

In order to compute the default time given by Equation (6) we have to compute the distribution function of \( X_1 \). As already mentioned, let \( F_{X_1} \) be the distribution function of \( X_1 \): this distribution function can be easily obtained from the Gamma distribution function:

\[
F_{X_1}(x) = P[X_1 \leq x] \\
= P[\sqrt{a} - G_1 \leq y] \\
= 1 - P[G_1 < \sqrt{a} - x] \\
= 1 - F_\Gamma(\sqrt{a} - x),
\]
\[ F_{\Gamma} \] is the distribution function of a Gamma\((a, \sqrt{a})\) random variable. In this case, Equation (6) to compute the default time \( \tau_i \) from the realizations \( A_i \) of the process \( A_i(t) \) becomes:

\[
\tau_i = p_i^{-1}(F_{\Gamma}(A_i)) = -\frac{\ln(F_{\Gamma}(\sqrt{a} - A_i))}{\lambda_i}.
\]

(10)

Without loss of generality, in the following we will consider \( a = 1 \).

### 3.3 Generating the empirical distribution of the portfolio loss

As already explained above, the aim of this paper is to design an effective DGS. In particular we want to define a target level for the funds to be collected by the scheme in order to promptly and effectively respond to financial crisis and protect the citizens. To achieve this objective, we have to build the distribution of the fund’s loss. The following hypotheses hold true:

- The time horizon \( T \) is 1 year.

- With reference to the DGS design outlined in the proposal by the European Commission (2010), we assume the DGS to have a target fund at its disposal equal to 2% of the amount of eligible deposits held by all banks joining the scheme.

- When a bank failure occurs, the fund has to pay back deposits of that bank. The exposure at default \( EAD_i \) is assumed to be equal to the amount of covered deposits held by that bank and thus the loss the fund suffers from is

\[
L_i = EAD_i \times (1 - 40\%).
\]

(11)

The total loss hitting the fund is estimated by aggregating individual bank losses.

The numerical simulation to build the empirical distribution of the fund’s loss is based on the following main steps:

1. Calibration of the default intensities \( \lambda_i \) by assuming all banks to have the risk-neutral default probability term structure described by Equation (1). From CDS spreads market data default intensities are estimated by inverting Equation (2). For all the other banks default intensities are calibrated from the default probabilities estimated from risk indicators, as detailed in Section 3.1.
2. Drawing of \(M\) realizations (one for each bank in the scheme) of the asset value process \(A_i(t)\), according to the models described in Section 3.2. Two different numerical simulations will be run; in the former, the process \(A_i(t)\) follows a one-factor Gaussian model, while in the latter \(A_i(t)\) follows a one-factor Shifted Gamma Lévy model.

3. Estimation of the default times \(\tau_i\) from Equations (8) and (10), using the \(A_i\) and \(\lambda_i\) obtained in step 2 and 1 respectively.

4. Simulation of the loss distribution: if \(\tau_i < T\), the bank defaults and the funds pays out an amount equal to \(L_i\) given by Equation (11).

In this exercise \(N = 100,000\), \(M = 51\) and \(\rho = 70\%\). Data on deposits are estimated from accounting data (Bankscope), Eurostat and from the data gathered from a survey distributed by the European Commission Joint Research Centre among European DGS in 2009. According to these data, the total amount of 2006 covered deposits in the sample is \(€277.4\) billion, the corresponding target size is around \(€7.7\) billion.
4 Results

In this section we want to present the results of the simulations described in Section 3. The main results are presented in Section 4.1. An additional analysis has been developed focusing on a legislative proposal adopted by the European Commission (see European Commission (2010)): results are presented in Section 4.2.

4.1 Simulations’ results

We have run the simulations $N = 100,000$ times and we have simulated the corresponding fund’s behavior. In the following we first present the results obtained when the banks’ underlying asset-value process follows a one-factor Gaussian model (Section 4.1.1) and then we move to the results corresponding to the one-factor Shifted Gamma Lévy model (Section 4.1.2).

4.1.1 Simulations’ results: one-factor Gaussian model

The two histograms in Figure 3 plot the empirical loss distributions of our reference banking system, which is the sample of 51 Italian banks, when the banks’ underlying asset-value process follows a one-factor Gaussian model. Figure 3(a) shows the empirical loss distribution of the whole sample of Italian banks. Figure 3(b) shows the conditional loss distribution of the sample: this is the loss distribution when at least one bank has failed and it has been built considering only the simulations containing at least one default.

(a) One year Italian sample banks’ loss distribution

(b) One year Italian sample banks’ conditional loss distribution

Figure 3: One year loss distribution of the Italian sample of banks; the banks’ underlying asset-value process follows a one-factor Gaussian model

Due to scaling problems we also report the corresponding loss distribu-
tions in Table 3. According to Table 3(a), the probability that at least one bank goes into default is equal to 4.15%. This probability has been computed by looking at the highest percentile in Table 3(a) where no default occurs.

(a) One year Italian sample banks’ loss distribution

<table>
<thead>
<tr>
<th>Percentile</th>
<th>Loss (million €)</th>
</tr>
</thead>
<tbody>
<tr>
<td>95.00%</td>
<td>0</td>
</tr>
<tr>
<td>95.85%</td>
<td>0</td>
</tr>
<tr>
<td>95.86%</td>
<td>2</td>
</tr>
<tr>
<td>97.65%</td>
<td>2,217</td>
</tr>
<tr>
<td>97.66%</td>
<td>2,319</td>
</tr>
<tr>
<td>98.00%</td>
<td>3,070</td>
</tr>
<tr>
<td>98.81%</td>
<td>7,645</td>
</tr>
<tr>
<td>98.82%</td>
<td>7,787</td>
</tr>
<tr>
<td>99.00%</td>
<td>13,003</td>
</tr>
<tr>
<td>99.90%</td>
<td>67,040</td>
</tr>
<tr>
<td>99.99%</td>
<td>127,014</td>
</tr>
<tr>
<td>100.00%</td>
<td>162,202</td>
</tr>
</tbody>
</table>

(b) One year Italian sample banks’ conditional loss distribution

<table>
<thead>
<tr>
<th>Percentile</th>
<th>Loss (million €)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25.0%</td>
<td>586</td>
</tr>
<tr>
<td>50.0%</td>
<td>2,873</td>
</tr>
<tr>
<td>71.3%</td>
<td>7,659</td>
</tr>
<tr>
<td>71.4%</td>
<td>7,723</td>
</tr>
<tr>
<td>75.0%</td>
<td>11,283</td>
</tr>
<tr>
<td>90.0%</td>
<td>30,911</td>
</tr>
<tr>
<td>95.0%</td>
<td>44,683</td>
</tr>
<tr>
<td>99.0%</td>
<td>91,913</td>
</tr>
<tr>
<td>99.9%</td>
<td>136,991</td>
</tr>
<tr>
<td>100.0%</td>
<td>162,202</td>
</tr>
</tbody>
</table>

Table 3: One year loss distribution of the Italian sample of banks; the banks’ underlying asset-value process follows a one-factor Gaussian model

We now can consider the effects of the protection afforded by the DGS, whose main aim is to absorb banks’ losses. In this analysis the scheme is assumed to have €7.7 (target fund) billion at its disposal. If we compare this amount with the loss distribution shown in Table 3(a), we can conclude that such a designed Italian DGS is able to cover up to 98.81% of its potential losses. The corresponding loss distributions are shown in Figure 4 and the figures corresponding to the most relevant percentiles are reported in Table 4: according to these figures, the probability that the DGS goes into default is around 1.19% (see Table 4(a)).

The simulation procedure described in Section 3 can be applied to seek the “optimal” size of the fund the DGS should set aside. First of all, the “optimal” criterion must be identified: one possible choice is to seek the optimal size of the fund, expressed as a percentage of eligible deposits, such that a desired percentage of the potential banks’ failures are covered. We have let the size of the fund vary over a wide range, i.e. from 1% to 50% and results are shown in Figure 5. The x-axis plots the target level’s sizes,

---

8Although ex-post financed, the Italian DGS has a virtual target fund at its disposal, whose size is equal to 0.8% of the amount of covered deposits (see Fondo Interbancario di Tutela dei Depositi (2006)). The virtual target fund, rescaled on our sample, is equal to €2.22 billion and it can cover up to 97.65% of the fund’s potential losses.
Figure 4: One year empirical loss distribution of the Italian DGS; the banks’ underlying asset-value process follows a one-factor Gaussian model

<table>
<thead>
<tr>
<th>Percentile</th>
<th>Loss (million €)</th>
</tr>
</thead>
<tbody>
<tr>
<td>95.00%</td>
<td>0</td>
</tr>
<tr>
<td>98.81%</td>
<td>0</td>
</tr>
<tr>
<td>98.82%</td>
<td>120</td>
</tr>
<tr>
<td>99.00%</td>
<td>5,335</td>
</tr>
<tr>
<td>99.90%</td>
<td>59,372</td>
</tr>
<tr>
<td>99.99%</td>
<td>119,347</td>
</tr>
<tr>
<td>100.00%</td>
<td>154,534</td>
</tr>
</tbody>
</table>

Table 4: Empirical loss distribution of the Italian DGS; the banks’ underlying asset-value process follows a one-factor Gaussian model expressed as a percentage of eligible deposits, while the y-axis plots the corresponding percentage of potential banks’ losses covered by the fund; the red point corresponds to the setting analyzed so far. If, for example, we want a target size which would cover at least 99% of the losses, the fund should set aside a fund equal to 3.2% of the amount of eligible deposits.

4.1.2 Simulations’ results: one-factor Shifted Gamma Lévy model

We now present the results when asset-value processes follow a one-factor Shifted Gamma Lévy model. The two histograms in Figure 6 plot the empirical loss distributions of our reference banking system. Figure 6(a) shows the empirical loss distribution of the whole sample of Italian banks, while
Figure 5: Optimum target; the banks’ underlying asset-value process follows a one-factor Gaussian model

Figure 6(b) shows the conditional loss distribution of the sample.

(a) One year Italian sample banks’ loss distribution
(b) One year Italian sample banks’ conditional loss distribution

Figure 6: One year loss distribution of the Italian sample of banks; the banks’ underlying asset-value process follows a one-factor Shifted Gamma Lévy model

Following the approach developed in Section 4.1.1, we report the corresponding loss distributions in Table 5. According to Table 5(a), the probability that at least one bank goes into default now slightly increase, from
4.15% of the previous simulation to 4.91%.

(a) One year Italian sample banks’ loss distribution

<table>
<thead>
<tr>
<th>Percentile</th>
<th>Loss (million €)</th>
</tr>
</thead>
<tbody>
<tr>
<td>95.00%</td>
<td>0</td>
</tr>
<tr>
<td>95.09%</td>
<td>0</td>
</tr>
<tr>
<td>95.10%</td>
<td>5</td>
</tr>
<tr>
<td>96.00%</td>
<td>313</td>
</tr>
<tr>
<td>98.00%</td>
<td>2,341</td>
</tr>
<tr>
<td>99.00%</td>
<td>6,080</td>
</tr>
<tr>
<td>99.17%</td>
<td>7,437</td>
</tr>
<tr>
<td>99.18%</td>
<td>7,750</td>
</tr>
<tr>
<td>99.90%</td>
<td>99,280</td>
</tr>
<tr>
<td>99.99%</td>
<td>166,440</td>
</tr>
<tr>
<td>100.00%</td>
<td>166,440</td>
</tr>
</tbody>
</table>

(b) One year Italian sample banks’ conditional loss distribution

<table>
<thead>
<tr>
<th>Percentile</th>
<th>Loss (million €)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25.00%</td>
<td>442</td>
</tr>
<tr>
<td>50.00%</td>
<td>1,046</td>
</tr>
<tr>
<td>75.00%</td>
<td>3,615</td>
</tr>
<tr>
<td>83.2%</td>
<td>7,642</td>
</tr>
<tr>
<td>83.3%</td>
<td>7,770</td>
</tr>
<tr>
<td>90.0%</td>
<td>16,689</td>
</tr>
<tr>
<td>95.0%</td>
<td>31,419</td>
</tr>
<tr>
<td>99.0%</td>
<td>164,455</td>
</tr>
<tr>
<td>99.9%</td>
<td>166,440</td>
</tr>
</tbody>
</table>

Table 5: One year loss distribution of the Italian sample of banks; the banks’ underlying asset-value process follows a one-factor Shifted Gamma Lévy model

We now consider the effects of the protection afforded by the DGS. If we compare the target at its disposal with the loss distribution shown in Table 5(a), we can conclude that such a designed Italian DGS is able to cover up to 99.17% of its potential losses. The corresponding loss distributions are shown in Figure 7 and the figures corresponding to the most relevant percentiles are reported in Table 6: according to these figures, the probability that the DGS goes into default is around 0.83% (see Table 6(a)).

(a) One year DGS loss distribution

<table>
<thead>
<tr>
<th>Percentile</th>
<th>Loss (million €)</th>
</tr>
</thead>
<tbody>
<tr>
<td>95.00%</td>
<td>0</td>
</tr>
<tr>
<td>99.00%</td>
<td>0</td>
</tr>
<tr>
<td>99.17%</td>
<td>0</td>
</tr>
<tr>
<td>99.18%</td>
<td>82</td>
</tr>
<tr>
<td>99.90%</td>
<td>91,612</td>
</tr>
<tr>
<td>99.99%</td>
<td>158,773</td>
</tr>
<tr>
<td>100.00%</td>
<td>158,773</td>
</tr>
</tbody>
</table>

(b) One year DGS conditional loss distribution

<table>
<thead>
<tr>
<th>Percentile</th>
<th>Loss (million €)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.00%</td>
<td>5,324</td>
</tr>
<tr>
<td>25.00%</td>
<td>8,370</td>
</tr>
<tr>
<td>50.00%</td>
<td>13,413</td>
</tr>
<tr>
<td>75.00%</td>
<td>28,470</td>
</tr>
<tr>
<td>90.00%</td>
<td>126,152</td>
</tr>
<tr>
<td>95.00%</td>
<td>156,835</td>
</tr>
<tr>
<td>99.00%</td>
<td>158,773</td>
</tr>
<tr>
<td>99.90%</td>
<td>158,773</td>
</tr>
<tr>
<td>100.00%</td>
<td>158,773</td>
</tr>
</tbody>
</table>

Table 6: Empirical loss distribution of the Italian DGS; the banks’ underlying asset-value process follows a one-factor Shifted Gamma Lévy model
Figure 7: One year empirical loss distribution of the Italian DGS; the banks’ underlying asset-value process follows a one-factor Shifted Gamma Lévy model.

If we consider the virtual target fund the Italian DGS has at its disposal (0.8% of covered deposits) and if we rescale it on our sample, we can conclude that such a fund can cover up to 97.93% of the potential losses.

We replicate the exercise described in Section 4.1.2 to seek the “optimal” size of the Fund the DGS should set aside. Results are shown in Figure 8.

Figure 8: Optimum target; the banks’ underlying asset-value process follows a one-factor Shifted Gamma Lévy model.
4.2 Simulation over 10 years

As already mentioned before, the European Commission adopted a proposal for a new Directive on DGS aiming at simplifying and harmonizing many aspects of the DGS functioning (see European Commission (2010)). According to this document, DGS should move to an ex-ante financing mechanism, protect only certain classes of deposits and the fund should reach within 10 years a target level equal to 2% of the amount of eligible deposits. Using the model described in Section 3, we have investigated the main features outlined in this document.

As already outlined, we have run the simulations $N = 100,000$ times and we have simulated the fund’s behavior over the transition period $T$ equal to 10 years. During this period, the size of the fund can fall below zero, but at the end of the transition period $T$ it must be positive; if not, the DGS is assumed to be in default. Figure 9 shows three possible fund’s paths: the green and the red paths represent the case where the scheme does not default, even if, in one case (red line) the fund falls below zero during the transition period; the blue path, on the contrary, represents the case where the scheme defaults, because at the end of the transition period the fund is negative.

![Figure 9: Simulated paths for the fund](image)

Following the approach of Section 4.1, we first present the banking system empirical loss distribution. The two histograms in Figure 10 plot the unconditional (Figure 10(a)) and conditional (Figure 10(b)) empirical loss distributions of our reference banking system, which is the sample of 51 Italian banks. Distributions are also reported in Table 7: according to the
We now can consider the effects of the protection afforded by the DGS, whose main aim is to absorb banks' losses. Recall that, during the transition period, the DGS collects annual contributions from its member banks such that, if no default occurs, the scheme is assumed to have €7.7 billion at its disposal at the end of the transition period. The fund’s loss distributions are shown in Figure 11 and the most relevant percentiles are reported in Table 8. According to these figures, the prob-
ability that the DGS goes into default is around 9.71% (see Table 8(a)).

![DGS loss distribution](image1.png) ![DGS conditional loss distribution](image2.png)

Figure 11: Empirical loss distribution of the Italian DGS

<table>
<thead>
<tr>
<th>Percentile</th>
<th>Loss (million €)</th>
</tr>
</thead>
<tbody>
<tr>
<td>90.00%</td>
<td>0</td>
</tr>
<tr>
<td>90.29%</td>
<td>0</td>
</tr>
<tr>
<td>90.30%</td>
<td>9</td>
</tr>
<tr>
<td>95.00%</td>
<td>18,747</td>
</tr>
<tr>
<td>99.00%</td>
<td>78,675</td>
</tr>
<tr>
<td>99.90%</td>
<td>147,212</td>
</tr>
<tr>
<td>99.99%</td>
<td>158,742</td>
</tr>
<tr>
<td>100.00%</td>
<td>158,773</td>
</tr>
</tbody>
</table>

Table 8: Empirical loss distribution of the Italian DGS

The simulation procedure described in Section 3 can again be applied to seek the “optimal” size of the fund the DGS should set aside. We have let the size of the fund vary over a wide range, i.e. from 1% to 50% and results are shown in Figure 12. If, for example, we want a target size which could cover up to 95% of the losses, the fund should fix a target level equal to 6.6% of eligible deposits.
Target as a % of eligible deposits

1−α

Target level of the Fund

90%
95%
99%
99.9%

Figure 12: Optimum target
5 Conclusions

This article has investigated a possible technique to design an effective DGS. We have focused in particular on the empirical loss distribution of the fund, as it can be used to assess the current level of security provided to deposits by the fund and to choose a proper target size for the fund. The empirical loss distribution has been gathered by simulating banks’ defaults and the corresponding losses potentially hitting the system.

The procedure adopted to simulate the loss distributions has relied on the classical credit risk techniques. Defaults have been assumed to occur if the bank’s asset-value has fallen below a threshold, asset-value processes have been assumed to follow a generic one-factor model and default times have been assumed to be exponentially distributed. A novel approach has been proposed to model asset-value processes and to estimate banks’ default probabilities. Asset-value processes have been assumed to follow a generic one-factor Lévy process and in particular the one-factor Gaussian model and the one-factor Shifted Gamma Lévy model have been investigated. Moreover, a novel approach has been proposed to estimate banks’ default probabilities, which are inferred from CDS spreads, assuming an underlying pricing model. In fact, CDS premia are regarded as among the best measures of the market pricing of credit risk currently available. This is mainly due to standardized contracts and the relatively high liquidity in the market. This procedure is quite common in literature, but to our knowledge it has never been explored in the context of DGS.

Our approach has been applied to a sample of Italian banks accounting for 60% of the amount of eligible deposits and for around 43% of total assets as of 2006. Moreover we have assumed the DGS to have at its disposal a target fund equal to 2% of the amount of eligible deposits. According to our results, such a designed DGS could cover up to 98.81% of its potential losses, in case the underlying asset-value process follows a one-factor Gaussian model. If the DGS wanted to set aside a fund capable to cover up to 99% of its potential losses, it should raise its fund up to 3.2% of the amount of eligible deposits. If we assume asset-value processes to follow a one-factor Shifted Gamma Lévy model, the corresponding DGS could cover losses up to 99.17% of its potential losses.
References


